Exercise 1 – Z notation (2/CS/3T)

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1 Logic and Quantifiers

N.B.: The general form for quantifiers in Z is: $\mathcal{Q}a : A \mid p \bullet q$
where $\mathcal{Q}$ is a quantifier (e.g., $\forall$ or $\exists$), a is the bound variable drawn from a set A, p is a constraint on a (expressed as a predicate), and q is the quantified predicate.
More that one declaration may be included: $a : A;\ b : B;\ \ldots$
For several declarations drawn from the same set use: $a, b, \ldots : A$
If the constraint is *true* it may be omitted: $\mathcal{Q}a : A \bullet q \ = \ \mathcal{Q}a : A \mid true \bullet q$
The set of natural numbers is denoted $\mathbb{N}$ and the set of integers is denoted $\mathbb{Z}$ in Z.

1. Specify the following in Z using quantifiers:

   (a) Every natural number greater than 10 is not equal to 5.

   (b) There exists some integer greater than 10 that is not equal to 5.

   (c) All natural numbers are also integers.

   (d) All even numbers are not odd numbers. (You may assume the presence of $even\,n$ and $odd\,n$ as predicates for even and odd numbers if you wish.)

   (e) Some students pass their exams.

   (f) All natural numbers when multiplied together give a result greater than or equal to both of the numbers.

2. Rewrite the following formulas in Z using quantifiers:

   (a) $10 > 2 \land 10 > 3 \land 10 > 4 \land 10 > 5$

   (b) $10 > 2 \lor 11 > 2 \lor 12 > 2 \lor 13 > 2 \lor 14 > 2$

   (c) $(10 > 2 \lor 11 > 2 \lor 12 > 2 \lor 13 > 2 \lor 14 > 2) \land$
   $(10 > 3 \lor 11 > 3 \lor 12 > 3 \lor 13 > 3 \lor 14 > 3) \land$
   $(10 > 4 \lor 11 > 4 \lor 12 > 4 \lor 13 > 4 \lor 14 > 4) \land$
   $(10 > 5 \lor 11 > 5 \lor 12 > 5 \lor 13 > 5 \lor 14 > 5)$

P.T.O.

2 Sets and Set Comprehension

N.B.: The standard form for set comprehension is: $\{a : A \mid p \bullet x\}$
where a is a variable declaration drawn from a set A, p is a constraint on a (expressed as a predicate), and x is an expression defining the set.
If x is formed from declaration, it may be omitted: $\{a : A \mid p\} = \{a : A \mid p \bullet a\}$
If the constraint is *true*, it may be omitted: $\{a : A \bullet x\} = \{a : A \mid true \bullet a\}$

1. Specify the following sets:

   (a) All natural numbers less than 5. Answer using:

      i. Set enumeration

      ii. Set comprehension

   (b) All even natural numbers

   (c) All odd natural numbers Answer using:

      i. Set comprehension

      ii. Set difference (using your answer for even numbers)

   (d) All squares of integers.

   (e) The natural numbers in terms of a restricted set of integers.

2. Specify the following using set operations such as $\subseteq$, $\cap$, $\cup$ and $\setminus$, naming and reusing previously defined sets if desired:

   (a) Natural numbers are also integers.

   (b) Even natural numbers and odd natural numbers are two distinct non-overlapping sets.

   (c) The combination of the even and the odd natural numbers makes up the complete set of natural numbers.

   (d) Some even natural numbers are not squares.

Note that information on the course is available on-line on the World Wide Web under the following URL (Uniform Resource Locator):

http://www.cs.reading.ac.uk/cs/people/jpb/teaching/z.html